JCRSCI | RESEARCH ARTICLE

On Naked Singularities of spacetime Curvature

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ABSTRACT

Considered Schwarzschild black hole solution within General Relativity. The area of black hole event horizon attracts stable and vivid interest. The literature brief review points in Introduction out, that the coordinate singularity on horizon might have physical meaning. In the following two sections this point strengthens. But this contradicts the dogmatic physical worldview, so in Discussion section I disarmed possible critics. I'm signing my faith in Conclusion, that this paper have found the naked (them we can observe) singularities. Even if is showed, that physically reasonable solutions can have the nakedness of singularities, they are far from the actual experimental detection. Contrary to that, recognizing the physical-ness of coordinate singularities makes them automatically experimentally verified. Because the event horizon is in definition of experimentally verified black holes (e.g., 2007 *MASTER* Team discovered the Black Hole *Ergosphere*, surrounding event horizon). **Context:** Natural Sciences: Physics: General Relativity theory: Black holes: event horizon. **Aims:** To publish original, innovative and novel research article resulting in contradicting results in the field of science. To publish theoretical paper that reports the contradictory finding (**coordinate singularity is actually real**) in scientific field. To present counter-examples (**singular force F and curvature scalar** ψ) ψ to usually accepted conjecture, which is "*coordinate singularity is unphysical*". **Settings and Design:** The singularity is defined in Introduction. There is set the function S(r), describing singularities. This is followed by derivation of gravitational attraction F. It turns to be singular with the S(r). Then directly from Curvature Tensor is derived singular scalar tensor ψ where you can see the S(r). Discussion answers possible critics preceded by Results. **Methods and Material:** Hardware: a PC; Software: a Web Browser, Maple V Release 5 with GRTensorII Version 1.70; Vector and Tensor analysis. **Results:** Although the surface gravity g has suffered from Scientific Community the procedure of Renormalization, nevertheless this operation left the gravitational attraction (force F) and scalar ψ to include singular function S(r). **Conclusions:** Already 11 years I firmly believe. That I found naked (one we can observe) singularity of Curvature, predicted by General Relativity. I called it "Sulo singularity". It's my personal joy and triumph, which I would like to share through JCRSCI.

Key words: coordinate singularity, curvature invariants, naked singularity, singularity, event horizon, black hole, general relativity, distributional geometry, gravitational collapse. **Key Messages:** Dear Scientific Community, to resolve contradictions and misconceptions, the Coordinate Singularity shall be renamed over to "Sulo singularity" or "Sulingularity". It is because, it's real (naked) singularity. And Sulo is my fallen father.

INTRODUCTION

What unites such titans of thoughts like Hawking and Susskind? The merciless battle[1] around the event horizon, which spitted out the *information loss paradox*. Where analogous paradoxes come in the first place from?

The coordinate singularity[2–6] on event horizon of a black hole has raised motivation in many minds since the discovery of Schwarzschild solution in 1916. Just like Twin Paradox in Special Relativity still trains the heads. Wikipedia 01.12.2011 article "Black Hole" refers to Gerard't Hooft 2009 lectures saying: *the singularity at the Schwarzschild radius* ($r_s \equiv 2M$ at event horizon) *an unphysical coordinate singularity*. Nevertheless couple years ago in rather extremely complicated way was claimed that the "horizon singularity" is not only a coordinate singularity without leaving Schwarzschild coordinates.[7] But I have critically examined this paper and calculated on my own, that his method artificially produced singular layer of exotic matter stuck to the horizon area. And exotic matter is ruled out by the positive energy condition[4] (whenever it's weak, strong, or null energy condition[4]). I found such fatal problem in other modern "Distributional Geometry" work.[8]

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DOI: 10.5530/jcrsci.2012.1.4

My neighbors, scientists from Tartu University also published conventional papers on horizon in highly cited journals.[9] From geodesic deviation (over first order of approximation) they found that the passage through the event horizon is locally detectable. This distantly indicates, that there could be something. First listed paper was the same year, as I completed their department with cum laude. Now permit me *to explore strange new worlds... to boldly go where no man has gone before.*[10]

Let's consider under singularity the extremely high finite values of a function near certain values of its argument, e.g. singularity of $S(r) \equiv (1-2M/r)^{-0.5}$ lies near $r = 2M$. Throughout the paper under S(r) is meant this functional dependence. I call a function "singular", if it has such values of argument. Spacetime "geometrized" mass is $M = G\,\mathcal{M}/c^2$, where $\mathcal{M}$ is mass in kilograms, G is gravitational constant.

Your *template* insists on asking *why the study was conducted.* To fight for the Truth in any aspect of my life, because Jesus Christ is **the truth** (John 14:6) being personalized. The truth is recognized as singular basis of our Civilization. You asking *the aims and objectives.* They fit the JCRSCI theme, I was said. For details refer to the above division "Aims:".

SUBJECTS AND METHODS

The force

The late 2011 Wikipedia elegantly wrote: "The **surface gravity**, g, of an astronomical or other object is the gravitational acceleration experienced at its surface. The acceleration of a test body at the event horizon of a black hole turns out to be infinite.. Because of this, a renormalized value is used." I disagree, that only because of very high values NEAR the horizon one can not use there conventional definition of surface gravity. Taking accelerometers the finite values would measure stationary observers hanging near the horizon. This corresponds to my singularity definition in Introduction.

I admit, that the renormalized surface gravity is not singular[4,5] and its dimension is acceleration. But I would like to find the force, measured in "Newtons", which would act on the bones of astronaut standing on some dense planet. Would they broke if one *thought experimentally* increases the density of the planet?

Three forms of the equivalence principle are in current use: weak (Galilean), Einsteinian, and strong.[11] *The weak equivalence principle... can be stated in many ways.*[11]

Beneficial for present paper is following.

The local effects of motion in a curved space (gravitation) are indistinguishable from those of an accelerated observer in flat space, without exception.[11]

This means, that seeing the accelerated fall of ball (g) in our cabin, we can not say whether cabin stands on dense planet, or is being accelerated by the rockets with g. Would it be the last case, the inertial force would have been simply m_o g. The m_o is the proper mass in Special Relativity, which is as known - invariant. *The equivalence principle guarantees that a gravity field (a central force) cannot be distinguished from forces due to uniform acceleration,*[11] thus the same valued force m_o g would experience astronaut on dense planet.

It's because the proper mass m_o is invariant also in General Relativity. Weren't it be the case, the proper energy 2 m_o c^2 of particle-antiparticle annihilation would differ on position of the free falling cabin. Which is forbidden by the strong equivalence principle: The outcome of any local experiment (gravitational or not) in a freely falling laboratory (cabin) is independent of the velocity of the laboratory and its location in spacetime.[11]

The proper velocity of a test particle measured by stationary observer is proper distance passed in proper time.[3] Taking the proper time derivative from proper velocity one gets the proper acceleration $g = c^2 M r^{-2} S(r)$.

The book[3] on the star solution shows, that Archimedes force compensating the gravity acting on small section of star is of form $F \sim r^{-2} S(r)$. The same form show references.[12] Wald is solidaristic: *of course, the locally exerted force.. becomes infinite on the horizon*, page 332.[4] I'm correcting him: *of course, the locally exerted force has singularity near the horizon*, according to the definition from my Introduction.

From the integral of motion[6] $m_o c^2 = mc^2 / S(r)$. The force would appear in the form $F = m_o g = m\kappa$, where is defined $\kappa \equiv c^2 M/r^2$. So the κ has dimension of acceleration. From mathematical concept of beauty (m_o g = mκ) it can be called the *renormalized surface gravity* of a dense planet of radius r. But the force F is still singular, because relativistic m is singular function (see Introduction). The observer in cabin can measure not relativistic m but proper m_o along with also proper g.

Note, that on pages 332, 158 in[4] the κ gained presentation as not local, measurable for infinite long weightless string. Where one sells one☺? So it is not local characteristic of localized thing - horizon! Thus, this case is in the Spirit of Contradictory Results (JCRSCI journal).

The curviness of spacetime drives all effects in General Relativity. It's described by Riemann curvature tensor. The singularity of the curvature tensor COMPONENTS is called coordinate singularity, because can be removed by coordinate transformation.[2–6] But I'm pointing your attention, that there's emissaries from curvature, that are invariant under coordinate transformations and showing singular behavior according to astronaut-observer. Like the blueshift of in-falling light, the proper acceleration g and force F, which broke the bones of astronaut. For him it doesn't matter, which coordinate system has described this deadly real force. In that view the $|g|$ and $|F|$ can be regarded as tensors. Scalars in particular.

I argue, that removing coordinate singularity from components of the tensor, doesn't remove it from the tensor itself. Because of following.

Singular curvature tensors

Using the Einstein summation rule, where any pair of indexes means summation from 1 to 4. Traditionally[4,3] the (fourth rank) tensor is being written as

$$\mathbf{R} = R_{\alpha\beta\eta\gamma}\mathbf{V}^{\alpha} \otimes \mathbf{V}^{\beta} \otimes \mathbf{V}^{\eta} \otimes \mathbf{V}^{\gamma}$$

where functions $R_{\alpha\beta\eta\gamma}$ are called "components", but the followings are "tensor products" of the basis vectors $\{\mathbf{V}^1, \mathbf{V}^2, \mathbf{V}^3, \mathbf{V}^4\}$. Components of the basis vector № 2, calculated in their own basis will be $V_i^2 = (0,1,0,0)$.

Basis vectors itself make the metric $\mathbf{V}^{\gamma} * \mathbf{V}^{\omega} = g^{\gamma\omega}$, where the scalar product (*) is also connected with metric, thus $g^{is} V_s^{\gamma} V_s^{\omega} = g^{\gamma\omega}$. This always holds, because basis vector components in their own basis are Kronecker deltas, i.e $V_s^{\gamma} = \delta_i^{\gamma}$.

Taking two basis vector fields $\mathbf{V}^1$ and $\mathbf{V}^3$. Multiplying basis vectors with curvature tensor we get function, which doesn't depend on the choice of metric (coordinates), i.e. it's true scalar

$$\begin{aligned}\Psi &\equiv \mathbf{R} * \mathbf{V}^3\, \mathbf{V}^1\, \mathbf{V}^3\, \mathbf{V}^1 \\ &= R^{ikgn} V_s^3\, V_{\_k}^1\, V_g^3\, V_n^1 = R^{ikgn}\delta^3_{\ i}\, \delta^1_{\ k}\delta^3_{\ g}\, \delta^1_{\ n} \\ &= R^{3131} = c^{\ 2} M r^{-5} (S(r))^2.\end{aligned}$$

It's large near horizon, where $r \approx rs$. Let's recheck. Perform the coordinate change $\{x^{\beta}\} \rightarrow \{w^{\varkappa}\}$. First coordinate system "A" and second system "B". Vectors $\mathbf{V}^1$ and $\mathbf{V}^3$ are two basis vectors from "A" and their components are Kronecker deltas. But they are not the basis vectors of the "B". So their components are different,[4] i. e. $(dx^{\beta}/dw^{\kappa})\delta^{\beta_3} = dx^3/dw^{\kappa}$ i.e. for vector $\mathbf{V}^3$. For transformation of covariant components R^{ikgn} one uses the opposite matrixes, i.e. (dw^{κ}/dx^{n}).

Thus, the invariant Ψ shows singular behavior of the compounds of the curvature tensor near the black hole horizon. Of course some quantities are not singular there. But at famous hypothetical singularity in the center ($r \approx 0$) not all quantities are singular. For example the scalar curvature $R_{\gamma\gamma}$ with concrete physical meaning.[13] It is zero everywhere,[3] also in the closest neighborhood of $r = 0$.

Any tensor (except zero rank) is determined not only by its components, but also by the basis vector fields. I argue, that if in certain coordinate system the components of curvature tensor are not singular, the basis takes the singularability over. One may be sure: the singular curvature could induce singular behavior of other locally detectable quantities. Some research group has published in Astronomische Nachrichten (highly cited, oldest astronomical journal), what the Kretschmann scalar can be singular near non-spherical horizon[14] with singular orbital acceleration even for spherical horizon.[14]

RESULTS AND CONCLUSION

Although the surface gravity g has suffered procedure of Renormalization, nevertheless this operation left the gravitational attraction (force F) and curvature scalar ψ to include singular function S(r).

Already 11 years I firmly believe, that I found the naked singularity of curvature,[15] predicted by General Relativity. It's my personal joy and triumph, I would like to share through JCRSCI. Because just two years ago Pankaj S Joshi has published: ....o*ne of the most important unsolved problems in astrophysics. Opening of naked singularities could change the search strategy unified theory of all physical interactions and not only because of the possibility of direct observational tests of this theory.*[15]

But we like remove unwanted things[2] and some people carrying them (e.g. this year I was dismissed from my hometown University) but for some true researcher they are still like splinter in the brain. In General Relativity all effects are driven by the curvature of spacetime and too many things are singular near event horizon. Like the redshift of light,[3] clocks slowing,[3] near light-speeds of massive test particles.[3]

DISCUSSION

An opponent might say: "in local coordinate frame (*tetrad*[4]) the Riemann tensor is not singular". I argue. Introducing coordinate frames, one changes the coordinate system. So this case belongs to Sec. 2.

An opponent might say: "the geodesic deviation equation[3,4] shows, that there is no catastrophic deformation of the in-falling body-astronaut passing the horizon. Where is the singularity, if it's real?". I argue. Such equation assumes, that proper time is absolute: if astronaut would compare the clocks attached to his head and legs, they always show the same time[3]. But as we know, the atomic clocks in building's basement are going slower, than at the roof. Then opponent might refer to the strong equivalence principle (Sec. 1). But this principle silently assumes, that the laboratory dimensions are negligible in respect to curvature. But this is not true in case, if the coordinate singularity is real. Just like we all were taught, that the central singularity ($r \approx 0$) rips apart the in-falling body by *tidal forces*.[4] Imagine, if you -*superhero*- standing on shore will throw a black hole into lake, you never see the waterline descending, because the forward front of water enters black hole on the bottom at infinite coordinate time.[3,5] But your hand-clock measures just this time.[3,5] But this is absurd: black holes "eat" the matter, even stars.[4] So the only logical solution is that super-strong gravitation of naked singularity compresses water near the horizon of black hole, becoming its assimilated part. Such innovative *tidal force is well defined*,[4] see page 68.

An opponent might say: "the geodesic trajectory of in-falling test particle can be mathematically continued inside the horizon. If there would be singularity, the geodesic were be terminated at the horizon, just like it does at the central singularity". I argue. Who can guarantee, that in infinite distance coordinate future (the Earth time) the area, where the spacetime metrics changes the signature (so the future-directed worldline tangent vector becomes past-directed[5]), it still has physical meaning? My own calculations show, that after test particle crosses the horizon, it leaves our Universe (the *coordinate map*[4]). It's because the Wronski determinant connecting *Schwarzschild coordinates* ↔ *Comoving coordinates of gravitational collapse* turns to un-allowed zero,[12] after the star surface drops down the horizon sphere. In a deeper sense the event horizon in General Relativity is the un-crossable light-speed barrier in Special Relativity. Because the principles of equivalence are foundations of Einstein's Relativity.

An opponent might say: "the geometrical nature of the General Relativity does not allow any force-interpretation". I disagree. The Curvature of Geometry is producing the value on Dynamometer, which designed to measure the forces. Otherwise the Newton would not introduced his *Inverse Square Law* $F \sim 1/r^2$. So the General Relativity shall not contradict the Classical Theory. I hope, with help of Contradictory Results (JCRSCI journal) this issue will be fixed as well. Wald is solidaristic: *we may meaningfully speak of the gravitational force field of the Earth*, page 68.[4]

An opponent might say: "there are papers you don't know, where is proved that there are no stationary observers in Schwarzschild spacetime. Especially near the horizon". I argue. First of all, the spacetime asymptotically becomes flat. And in flat spacetime certainly can exist observers with stationary position. Secondly, if even in principle, even *thought experimentally* [4, page 158] the observer can not hold its position (even for a second), then it's the additional proof. That there is the real singularity. Where physical and philosophical concepts like time, energy, stationarity loose their common sense.

ACKNOWLEDGEMENT

To my mother V*alentina A Martila*, for emotional support.

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